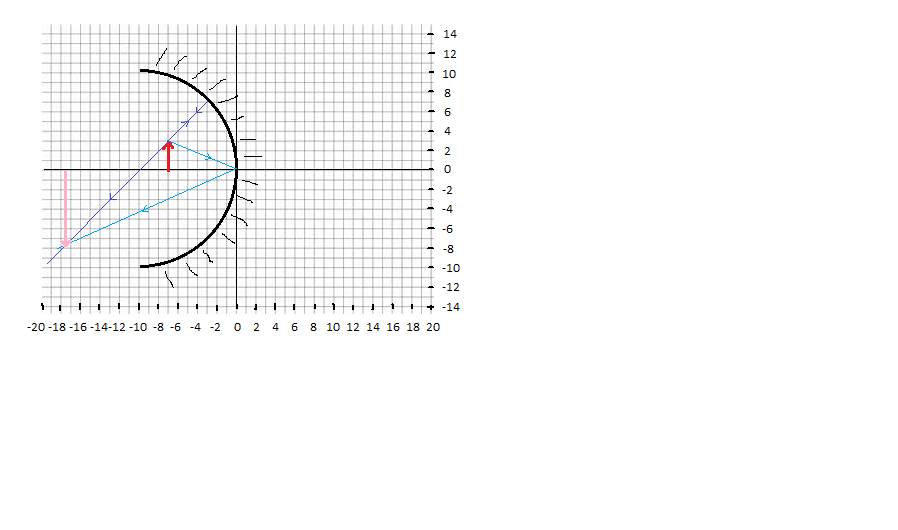
**Homework 6 Solutions Due 3/2**

\* For problems 1-8, I’d like/want/demand to see the rays used to locate the image.

**Problem 1.** Consider the object below, in front of the concave mirror. (a) Using ray tracing alone, determine the location and size of the image. Let one ray proceed from the tip of the object to the center of the mirror on the principal axis, and the other proceed from the tip of the object directly away the center *of curvature* of the mirror. Fill your results into the table, with proper signs and all.

****

|  |  |
| --- | --- |
| **xi** | 17.4 |
| **hi** | -7.7 |

(b) Now determine the position/height using the mirror equations. Are the results close? They *should* be, because the mirror equation is exactly true – just saying.

According to these we’d have:

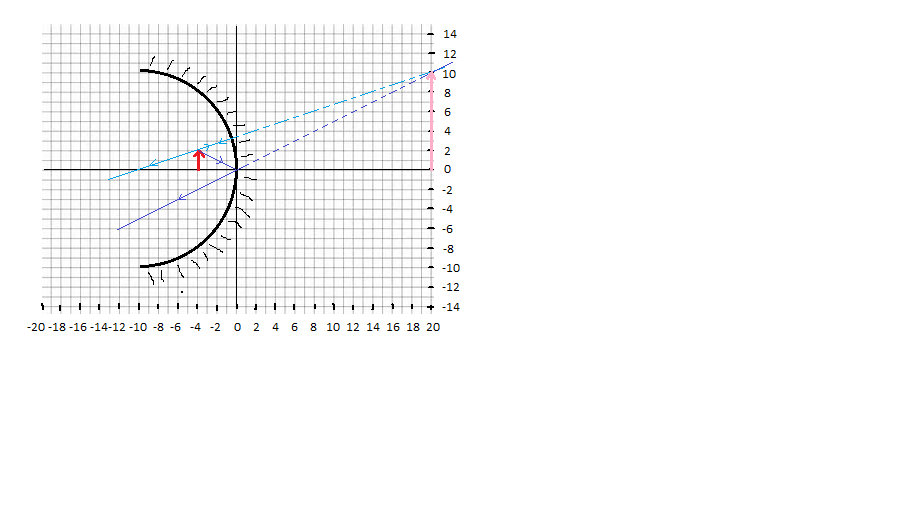


And,



Seems pretty good.

**Problem 2.** Consider the object below, in front of the concave mirror. (a) Using ray tracing alone, determine the location and size of the image. Let one ray proceed from the tip of the object to the center of the mirror on the principal axis, and the other proceed from the tip of the object directly away the center *of curvature* of the mirror. Fill your results into the table, with proper signs and all.

****

|  |  |
| --- | --- |
| **xi** | -20 |
| **hi** | 10 |

(b) Now determine the position, height using the mirror equations. Are the results close?

We have:

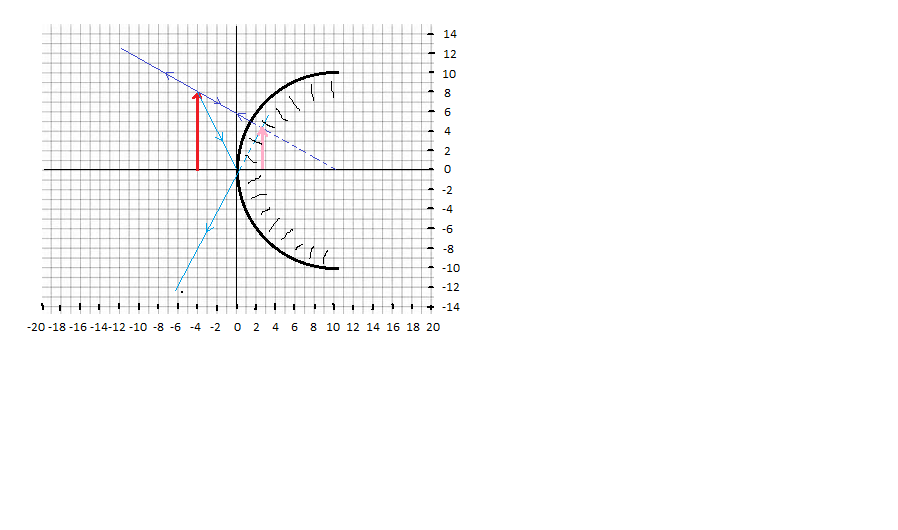


And,



Also good agreement.

**Problem 3.** Consider the object below, in front of the concave mirror. (a) Using ray tracing alone, determine the location and size of the image. Let one ray proceed from the tip of the object to the center of the mirror on the principal axis, and the other proceed from the tip of the object directly towards the center *of curvature* of the mirror. Fill your results into the table, with proper signs and all.

****

|  |  |  |
| --- | --- | --- |
| **xi** |  | -2.8 |
| **hi** |  | 4.2 |

(b) Now determine the position, height using the mirror equations. Are the results close?

Again,



And,



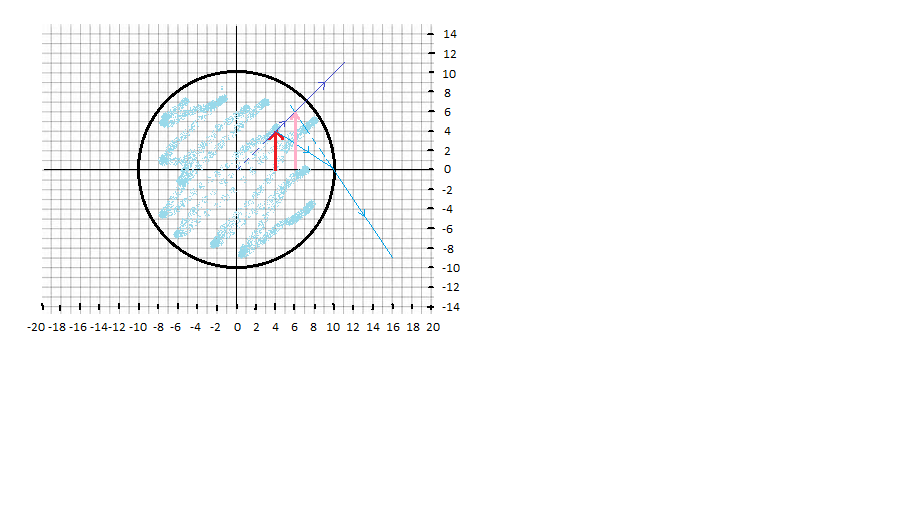
Pretty good again.

**Problem 4.** Now consider the object below in a refractive medium, like a fish bowl of water, but with an index of refraction n = 1.5 rather than 1.33. (a) Using ray tracing alone, determine the location and size of the image (as viewed from outside the medium, on the right looking left). Let one ray proceed from the tip of the object to the center of the interface on the principal axis, and the other proceed from the tip of the object directly away the center of curvature of the interface. Note you’ll have to use Snell’s law to do this correctlly. Fill your results into the table, with proper signs and all.

Drawing rays below. The ray incident on the principle axis makes an angle θ = tan-1(4/6) = 33.7°. And so by Snell’s law, the refracting angle would be:



Of course we must trace the rays backwards to obtain the location of the image.

****

|  |  |
| --- | --- |
| **xi** | -4 |
| **hi** | 6 |

(b) Now determine the position, height using the refractive surface equations. Are the results close? Note that the refractive surface equations explicitly invoke a small angle approximation in their derivation, which our object violates. So don’t expect *too* much accuracy.

doot. de-doot. de-doo



And height,



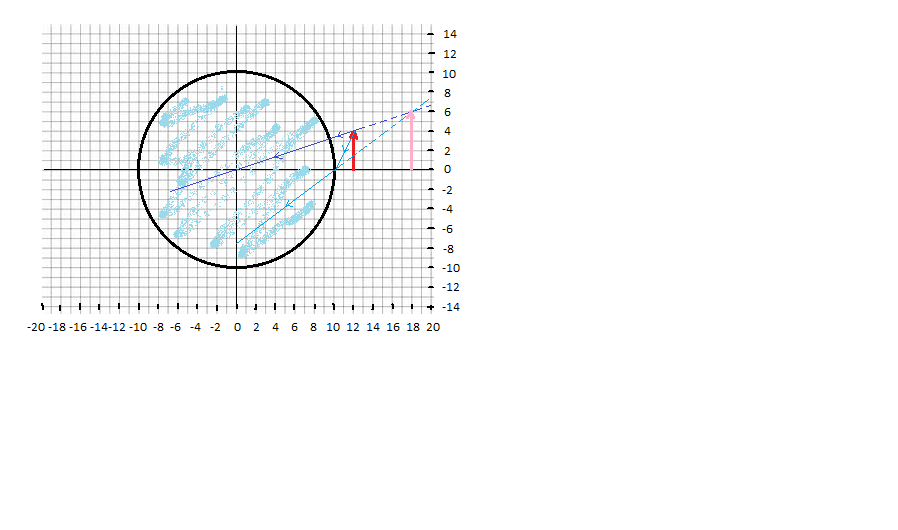
This is off a little bit.

**Problem 5.** Now consider an object outside the n = 1.5 water filled fish bowl, but in the n = 1 air. From the fish’s perspective, looking right, what would be position and location of the person (object) shown below. Again, use a ray that heads towards the center of curvature of the interface, and one that hits the center of the interface on the principal axis. And again, you’ll have to use Snell’s law. Fill your results into the table, with proper signs and all.

Drawing rays below. The ray incident on the principle axis makes an angle θ = tan-1(4/2) = 63.4°. And so by Snell’s law, the refracting angle would be:



Again we must trace the rays backwards to obtain the location of the image.

****

|  |  |
| --- | --- |
| **xi** | -8 |
| **hi** | 6 |

(b) Now determine the position, height using the refractive surface equations. How do these results compare. And once again, don’t expect great agreement since the small angle approximation is being violated.

So,



And height,



These results are quite off too. Again, the limitations of the small angle approximation show up.

**Problem 6.** Think we’re done? Foolish person. We have not yet begun to physics! Let’s consider a semi-circular shaped lens – the circular part has radius of curvature |R| = 5. Let it have an index of refraction n = 1.5 too. (a) Use Snell’s law to determine the location of the image. The two rays you’ll want to use are these: one which proceeds from the tip of the object parallel to the principal axis so as to hit the flat surface perpendicularly, and another which emerges from the tip and impinges upon the flat surface at its center, where it intersects the principal axis. Fill your results into the table, with proper signs and all. You can measure distance from the flat surface.

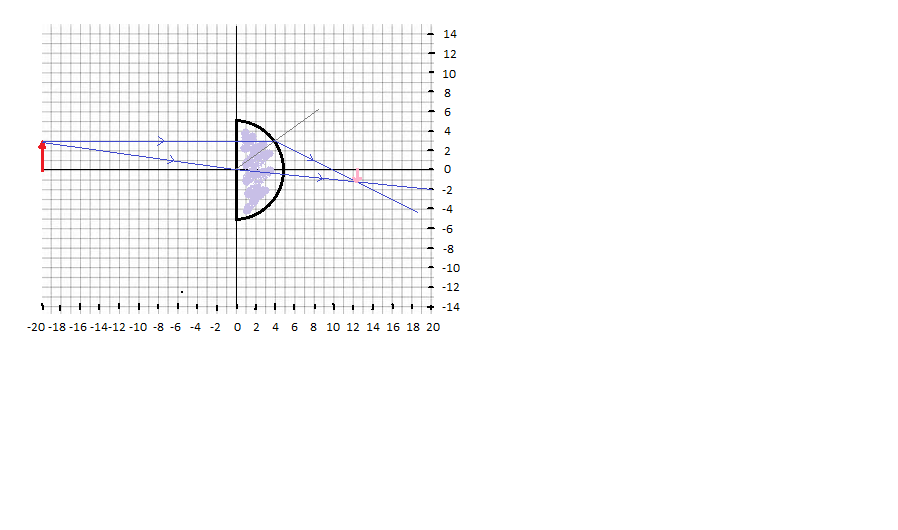
So the ray incident on the principal axis makes an angle θ = tan-1(3/20) = 8.5°. And the angle it makes in the lens will be:



This ray will hit the curved part of the lens perpendicularly (since it is originating from the center of curvature of that surface) and so it will not bend when it leaves the curved interface. The top ray, on the other hand, will not refract when it hits the flat side of course, but it will when it hits the curved part. The normal line is drawn, and the incident angle the ray makes is sin-1(3/5) = 36.9°. And so the refracted angle will be:



These two rays will intersect to form a real image, as shown below:

****

Our results are:

|  |  |
| --- | --- |
| **xi** | 12.5 |
| **hi** | -1 |

(b) Compare this to your results using the lens equation. The comparison won’t be too favorable because the lens equation requires the lens to be thin, which ours is *not*.



And,



And as we see, not great agreement.

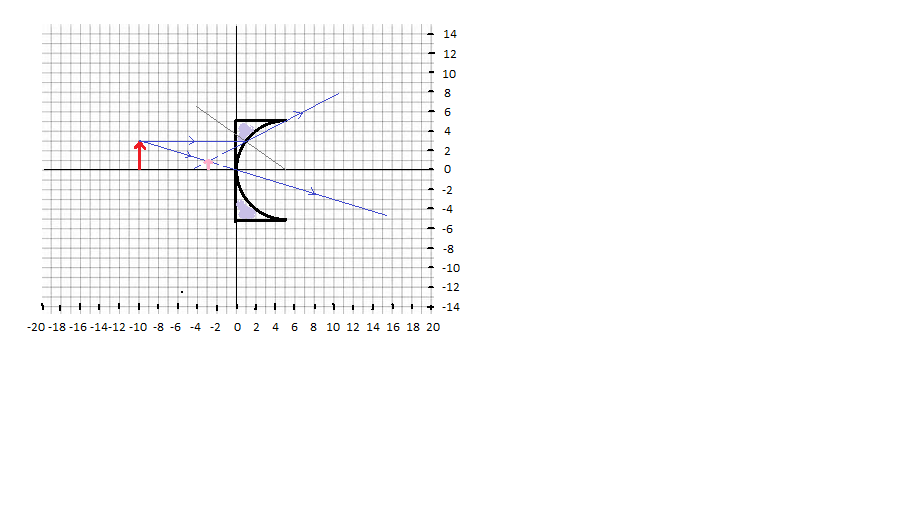
**Problem 7.** Last one (kind of)! Let’s consider diverging lens, with the circular part radius of curvature |R| = 5. Let it have an index of refraction n = 1.5 like before. (a) Use Snell’s law to determine the location of the image. Once again, the two rays you’ll want to use are these: one which proceeds from tip of the object parallel to the principal axis so as to hit the flat surface perpendicularly, and another which proceeds to the center of the flat surface, where it intersects the principal axis (this ray will not refract since the width of the lens at this point is zero). Also, I’ve drawn that line emanating from the center of curvature to help a bit with applying Snell’s law. Fill your results into the table, with proper signs and all. You can measure distance from the flat surface.

So the ray that’s incident on the principal axis will not refract at all, because the thickness of the lens at that point is technically zero. The ray parallel to the principal axis will not refract when it hits the flat surface but will when it hits the curved surface. The angle it makes w/r to the normal line is

θ = sin-1(3/5) = 36.9°. The angle it’ll make when it refracts is:



Just like before. But now the rays are separating, and will only ‘intersect’ if traced backwards. The virtual image formed is illustrated.

****

Our results are:

|  |  |
| --- | --- |
| **xi** | -3 |
| **hi** | 1 |

(b) And compare to the lens equations. Results will be a little better since the lens is *thinner*.



And,



And as we see, not great agreement, but better than last time.

**Problem 8.**  Consider the side-mirrors on a car. These are convex mirrors.

(a) List some reasons why that should be so.

Because image distance is always negative (behind the mirror) where you can see it.

It always makes images smaller, so you can see all of the object inside the mirror, not just parts of it.

Such a mirror has a wider field of view, since it curves outwards.

(b) These mirrors always say, ‘object is closer than it appears’. Could this be a reference to the image position being further from the mirror than the object position? Maybe/Maybe not. Which is larger for a convex mirror: xo or xi? Prove it.

xo is always larger, because



Since f is negative, the denominator of the fraction will always be greater than the numerator and so the (absolute value of the) fraction will always be less than 1. So xi < xo.

(c) What about image size? Which is larger for a convex mirror: ho or hi? Prove it.

Apropos height we have:

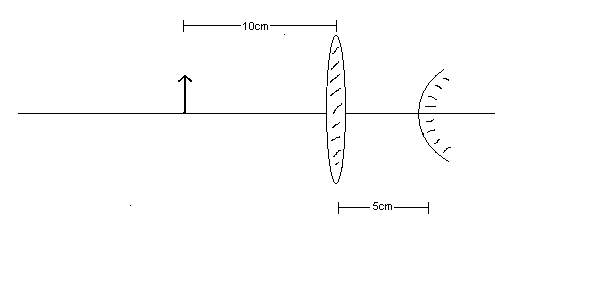


This follows since |xi| < |xo| → the absolute value of the magnification must be less than 1.

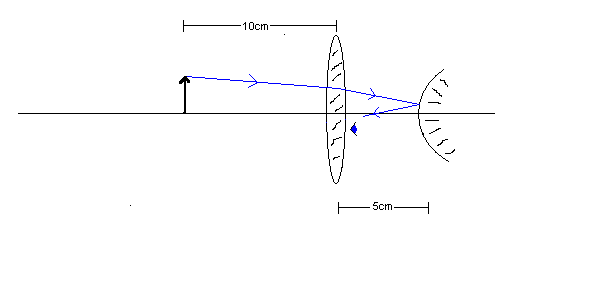
(d) Do you have a conjecture for why the ‘object is closer than it appears’?

It isn’t because xo < xi, that’s for sure. Rather it is because the image appears so much smaller than the object, and we generally associate small with far away.

**Problem 9.** Suppose you have a 15cm tall object 10cm away from a convex lens (focal length 7cm), which itself is 5cm away from a convex mirror (radius of curvature 5cm). (a) Locate the image you would see if you were standing between the lens and the mirror, looking towards the mirror – specify its position with respect to the mirror (left/right).



If you are standing in between the lens and mirror, then the image you see will come from the light that goes through the lens, reflects off of the mirror. Pictured below is the path of such a typical light ray:



We shall proceed in 2 steps. We’ll find the image produced by the lens when the light goes through it the first time. Then we’ll see what image the mirror produces when it reflects this light. So we proceed with the lens equation,



So the image that would be produced by the lens (were the mirror not there) is 23.3cm beyond the lens, which places it 18.3cm beyond the mirror. The magnification of the image is,



We treat this first image formed by the lens as the object for the mirror. Since the first image landed 18.3cm beyond the mirror (in the region where light is *not* when it hits the mirror), the object distance is considered to be *negative* 18.3cm. Also recall the focal length for the convex mirror is -R/2. So we have,



And the magnification is:



Since xi2 = -2.9, the image formed by the mirror is located 2.9 cm *to the right of* the mirror.

(b) What is the height of this image?

Height is given by the overall magnification:



(c) Is the image real or virtual?

Since we have a negative image distance, the image is virtual.

**Problem 10.** A nearsighted person can focus on objects as close as 10cm, but can’t see objects clearly when they are beyond 1.5m.

(a) What focal length should the prescribed contact lens have to correct this problem? What kind of lens is it?

So the far point of the person is 1.5m. So we need to take objects at infinity and focus them at xi = -1.5m. So we have,



So the near sighted person needs a diverging lens as expected.

(b) What is the new near point?

The new near point is the closest an object can be so that he can still see it. So now we use the lens equation and determine where an object has to be for its image to be focused at -10cm.



So the new near point is at 10.7cm – farther away than before as expected, but not by much.

**Problem 11**. Gandalf is farsighted, both metaphorically and actually. His near point is 40cm. What lens power does Gandalf prescribe himself so that he can see clearly at x = 25cm? Also, draw a ray diagram illustrating the object and its image, as focused by the lens.

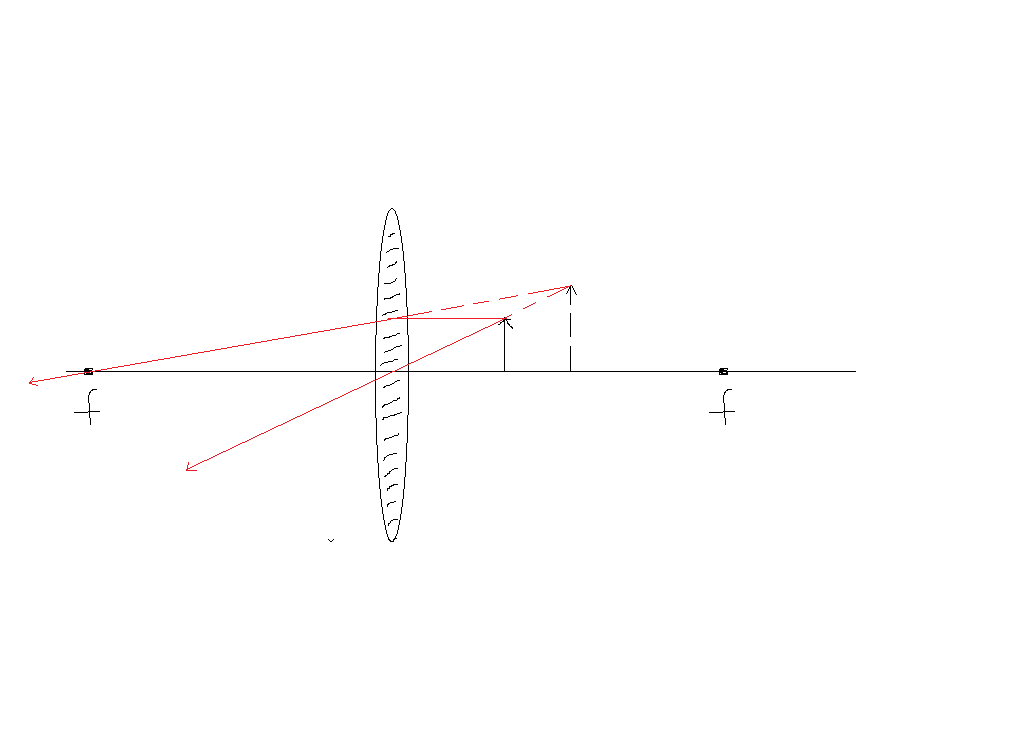
We need lens to project an image of object at 25cm to 40cm. So,



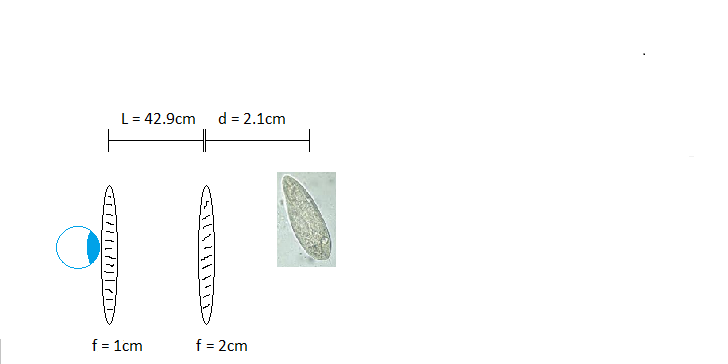
The corresponding power is:



and diagram looks like,



**Problem 12.** Say hi to Pete the paramecium, sitting under a microscope. What angular magnification does the microscope impart to his image? Assume a standard nearpoint of x = 25cm. Also compare to the standard magnification formula.



So, working out where the first image will be:



and the height of the image will be:



The position of the second image is:



and the height of the second image will be:



the angular size of the image will be:



The angular size of the object, at the near point is:



So the magnification is:

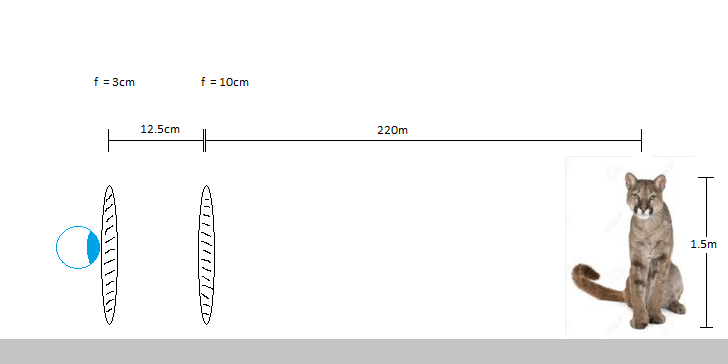


The approximate formula gives:



which is pretty close.

**Problem 13.** You are walking down the street when you think you see a 1.5m tall puma. Just to be sure, you pop the lenses out of your glasses and construct a telescope. What is the angular magnification of the puma? Compare to the standard approximate formula.



The angular size of the puma, without lenses, is:



Now let’s see where and how large the image will be. So second (f1) lens will create image of puma at:



to the left. And the height of the image will be:



And then this will be the object for the second lens a distance 2.5cm away from it, which will create an image at:



with a height,



So the angular size of the image will be:



which is an angular magnification of:



The approximate formula is:

